Bending a soccer ball with math

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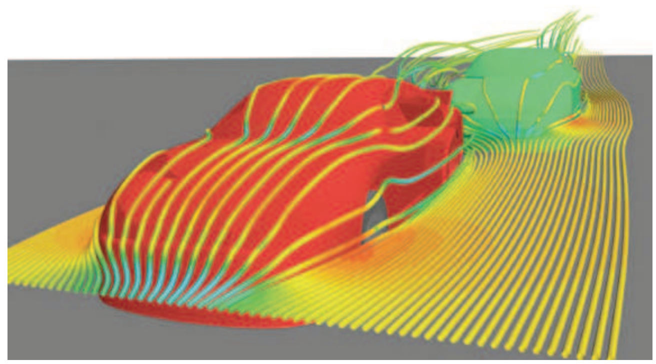
Aerodynamics in sports has been studied ever since Newton commented on the deviation of a tennis ball in his paper *New theory of light and colours* published in 1672. Today, the field of computational fluid dynamics (CFD) studies the effect of aerodynamics in such sports as soccer and NASCAR racing. See Figure 1.

(a)



(b)

Figure 1: CFD studies aerodynamics in sports. In (a), CFD research predicts the fight of a soccer ball. In (b), a simulation of two NASCAR cars visualizes the streamlines of air produced as a car drafts and is about to pass another.

Soccer matches are filled with complex aerodynamics as evidenced in the way balls curve and swerve through the air. World class soccer players such as Brazil's Roberto Carlos, Germany's Michael Ballack, and England's David Beckham exploit such behavior, especially in a free kick.

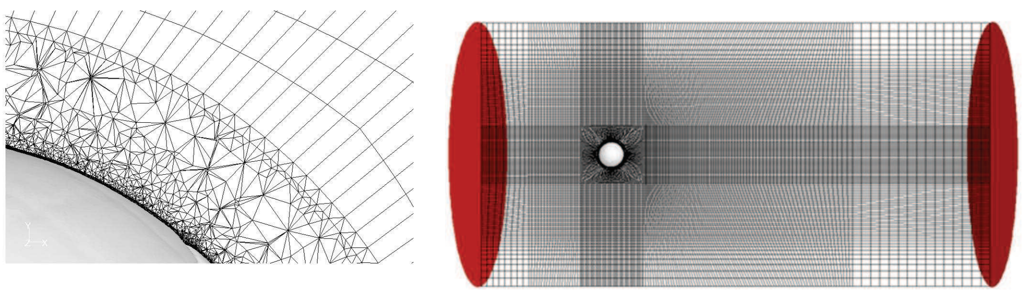
According to research by the University of Sheffield's Sports Engineering Research Group and Fluent Europe Ltd., the shape and surface of a soccer ball, as well as its initial orientation, play a fundamental role in its trajectory. CFD research has increased the understanding of the flight of a knuckleball, which is kicked as to minimize the spin of the ball and to confuse a goalkeeper. The research group focused on shots resulting from free kicks, in which the ball is placed on the ground after a foul, for instance.

Calculating the trajectories of objects is a common problem in calculus where the absence of air resistance is generally assumed. Drag forces affect the path of a soccer ball and are of two main types: skin friction drag and pressure drag. Skin friction drag occurs when air molecules adhere to the surface of the ball, which results in friction from the interaction of the two bodies. Pressure drag occurs when the air reaches the rear of the ball. A large area then opens up for the airflow. Since the amount of moving air per unit area must be constant because we are not adding or removing air the flow must slow down. Separation occurs when the air slows down so much that it is not moving or even moving backwards, which results in a wake as seen behind moving boats.

A soccer ball has a steep surface which results in a large wake; pressure drag dominates. The body of the racing car in Figure 1 (b) is streamlined and has less pressure drag. So, friction drag dominates.

Laminar flow occurs when streams of air flow in parallel layers. Turbulent flow is characterized by chaotic disruption between layers. Laminar flow is seen in Figure 1 (b) toward the front of the lead car. Turbulent flow occurs between the cars and is less visible in the picture. Both flows affect the trajectory of a soccer ball.

A turbulent boundary layer mixes air flows producing more energy close to the soccer ball. The turbulent boundary will cling to the surface longer and the ball will have a smaller wake. For soccer balls, a turbulent boundary layer gives a lower total drag than a laminar boundary layer. So, a transition to laminar airflow causes a soccer ball to slow down quite suddenly and potentially dip in its trajectory. The seams of a soccer ball cause more turbulence than would a perfectly smooth sphere with no seams.



(a) (b)

Figure 2: An important step in CFD simulations is capturing the geometry of a soccer ball with a 3D non-contact laser scanner. The mesh in (a) has approximately 9 million cells. As seen in (b), the space around the ball is meshed as to determine the flow of air.

With the surface so critical, an important step in CFD simulations of soccer balls is capturing the geometry of the ball with a 3D non-contact laser scanner. The mesh digitizes the surface of the ball down to its stitching as seen in Figure 2 (a). The refinement near the seams is required to model properly the path of air near the ball's surface. Since 1970, the official tournament ball of the World Cup has been produced by adidas for the competition, which is held every four years. From the tournament balls, four balls with different panel designs were selected to be scanned. Among the digitized balls was the adidas Teamgeist ball used in the 2006 World Cup.

Some free kicks in soccer have an initial velocity of almost 70 mph. Wind tunnel experiments demonstrated that air near the ball's surface changes from laminar to turbulent flow at speeds between 20 and 30 mph, depending on the ball's surface structure and texture.

Mathematically, the governing equations in such a simulation are the Navier-Stokes equations, which are based on the (a) the conservation of mass, (b) conservation of momentum and (c) conservation of energy. The research on high-velocity, low-spin kicks did not employ (c) since an isothermal flow was assumed, meaning that the flow remained at the same temperature. The flow was also assumed to be incompressible and Newtonian. While air is a compressible fluid, this becomes influential only if a ball travels over Mach 0.3 or about 111 yards per second! A Newtonian fluid's viscosity depends only on temperature. Honey is Newtonian; if you warm it up, its viscosity decreases and it flows more easily. The viscosity of non-Newtonian fluids like shampoo or pudding depends on the force applied to it or how fast an object moves through the liquid.

These assumptions allow simplification of the Navier-Stokes equations. We again have conservation of mass in which the divergence of velocity is zero. Stated in vector form, $\nabla \cdot \overline{v} = 0$. Conservation of momentum follows:

$$\rho\left(\frac{\partial \overline{v}}{\partial t} + \overline{v} \cdot \nabla \overline{v}\right) = -\nabla p + \mu \nabla^2 \overline{v} + \overline{f}.$$

In this equation, the term on the left-hand side describes the inertia of the flow. The right-hand side of the equation is the sum of several forces. First, ∇p represents the pressure gradient, which is a physical quantity describing in which direction and at what rate the pressure changes most rapidly around a particular location. It arises from forces applied perpendicularly to the soccer ball. The second term, $\mu \nabla^2 \overline{v}$, represents the viscous shear forces in the air, which are tangential to the soccer ball. Finally, $\overline{f}$ represents other forces, usually gravity.

The techniques developed in Sheffield made possible a detailed analysis of the memorable goal scored by David Beckham of England in a match against Greece during the World Cup Qualifiers in 2001. A foul on an English player resulted in a free kick at a distance of about 29 yards from the goal. A group of defenders, the defensive wall, stood side by side on the field between the ball and Greece's goal. Beckham's shot left his foot at about 80 mph. The ball cleared the defensive wall by about one and a half feet while rising over the height of the goal. At the end of its flight, it slowed to 42 mph and dipped into the corner of the net. Calculations showed that the flow around the ball changed from turbulent to laminar flow several yards from the goal. If it had not, the ball would have missed the net and gone over the goal's crossbar.

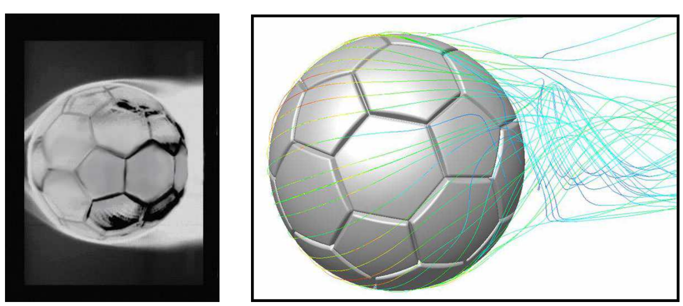


Figure 3: (left) Wind tunnel smoke test of a non-spinning soccer ball. (right) CFD simulation showing wake flow pathlines of a non-spinning soccer ball, air speed of 27 mph.

In a sense, Beckham's kick applied sophisticated physics. Our understanding of these dynamics could affect soccer players from beginner to professional. For instance, ball manufacturers could produce a more consistent or interesting ball that could be tailored to the needs and levels of players. Such work could also impact the training of players. Among the researchers on this project was Sarah Barber who commented, "As a soccer player, I feel this research is invaluable in order for players to be able to optimize their kicking strategies."

To this end, there is a simulation program called Soccer Sim developed at the University of Sheffield. It predicts the flight of a ball given input conditions that can be acquired from the CFD and wind tunnel tests, as well as from high speed videos of players' kicks. The software can be used to compare the trajectory of a ball given varying initial orientations of the ball or different spins induced by the kick. Moreover, the trajectory can be compared for different soccer balls.

Analyzing aerodynamics in sports can increase the speed of a bicyclist or bobsledder or produce a more effective fastball or free kick. In CFD research, much of the work is conducted without the presence of an athlete. The impact of the CFD research of soccer balls will be seen over time and may give more insight on how to bend a soccer ball – regardless of and possibly due to its design.

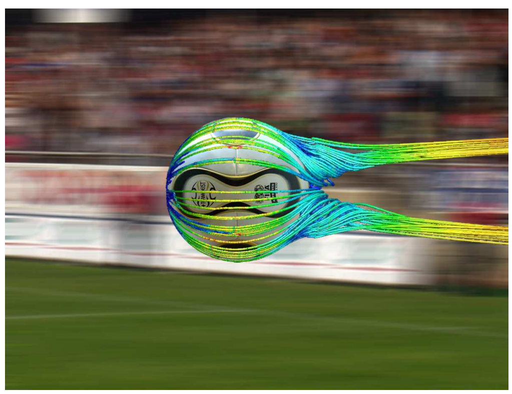


Figure 4: High speed airflow pathlines colored by local velocity over the 2006 Teamgeist soccer ball.

Acknowledgements: Images courtesy of Fluent Inc. and the University of Sheffield. The author also thanks Sarah Barber for her help on this article.

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About the Author

TIMOTHY P. CHARTIER is an Associate Professor of Mathematics at Davidson College. He is a recipient of the Henry L. Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member from the Mathematical Association of America. As a researcher, Tim has worked with both the Lawrence Livermore and Los Alamos National Laboratories on the development and analysis of computational methods to increase the efficiency and robustness of numerical simulation on the lab's supercomputers, which are among the fastest in the world. Tim's research with and beyond the labs was recognized with an Alfred P. Sloan Research Fellowship. In his time apart from academia, Tim enjoys the performing arts, mountain biking, nature walks and hikes, and spending time with his wife and two children.

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