Surprising Streaks and Playoff Parity: Probability Problems in a Sports Context

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Abstract

Sporting events generate many interesting probability questions. In probability counterintuitive results are common and even experienced mathematicians can make mistakes in problems that sound simple on the surface. In this article we examine two sports applications in which probability questions have results that may come as a surprise to sports fans and mathematicians alike. The first concerns the chances of rare events in sporting contests, with recent examples from baseball and football. The second considers the merits of longer playoff series in sports like major league baseball and professional basketball, where a set of games is used to determine who advances in a post-season tournament.

Problem 1: Rare Events

One of the most appealing aspects of watching sports events is the possibility that a viewer may see something especially noteworthy, perhaps even unprecedented. Sports fans who have the good fortune to be in attendance when something spectacular happens have vivid memories and wonderful stories. Baseball fans love to describe the time they saw a no-hitter, a player hit for the cycle, a triple play, or some other rare event. A typical response to such descriptions is, "Wow, what are the chances of that?"

Those of us who love sports and math frequently are asked, "What are the chances of …?" It may be easy to come up with a quick estimate of the probability, but first guesses are often wrong, sometimes off by orders of magnitude! Making estimates of the probability of rare events can be difficult. Checking the answer may be hard because the events are sometimes so unusual that we often do not have enough historical instances to get a sense of the probability. A problem of this type for sports fans occurs in the first round of the annual Division I men's basketball tournament of the National Collegiate Athletic Association. Since 1985 when the tournament field expanded to 64 (now 65) teams, a 16th seed has yet to beat a top-ranked first seed. The probability of a 16th seed winning a first round game is not zero, but it is small and we have no past history to inform us.

We look at examples of streaks in a baseball game and a football season as a way to explain some of the difficulties that occur when thinking about rare events.

Example 1: Four homers in a row

On April 22, 2007 the Boston Red Sox had a remarkable third inning against the New York Yankees. Red Sox slugger Manny Ramirez came to the plate with two out and nobody on and launched a home run. Then J. D. Drew, Mike Lowell, and Jason Varitek followed Ramirez and each of them hit a home run. Such a back to back to back to back event is rare indeed as this was only the fifth time in major league history that it had happened. And so on April 23 the always lively sports media conversations around Boston centered on it and the natural question was, "What are the chances of that?"

Many people were glad to try to answer. A Boston Globe sports reporter got estimates from several people with good credentials for this sort of work: a Red Sox employee, an academic from California who had looked into the question when the Los Angeles Dodgers hit four home runs in a row in a previous year, and a mathematics professor at one of the many colleges in the Boston area. (Author's note: It wasn't me!) According to the article one of the experts said:

> "What is needed in finding the various probabilities that you are interested in is the following ratio: *p*=the total number of home runs hit in the major leagues, divided by the number of plate appearances. … It would be reasonable … to use the numbers from 2006 to come up with that number. Last season, there were 5,386 home runs hit in 188,052 plate appearances. Thus *p* is equal to .02857. The probability of four consecutive home runs is p to the fourth power, or *p* times *p* times *p* times *p*. That equals .000000673, which in this case means there is a one in 1.4 million chance."

There is a problem with that estimate. In the approximately 170,000 major league baseball games that have taken place since 1900 the event "four home runs in a row" has happened five times. So history suggests a frequency like one in 34,000 rather than one in 1.4 million! What's going on?

Let's call the "1 in 1.4 million chance" our first estimate. Any baseball fan with a little bit of probability knowledge sees one problem and one questionable assumption in this estimate. The questionable assumption is that raising *p* to the fourth power assumes that the four events are independent, which may not be so. Baseball old-timers, for instance, would claim that the probability of a batter being hit by pitch might go up significantly after two or three homers in a row, which would make the event less likely. Others would argue that seeing two or three homers in a row suggests that the pitcher or pitchers are struggling, which would make the event more likely. We retain the independence assumption for now because correcting it would require an empirical study of past games that is not our primary concern.

The problem is that the four hitters in our sequence are all power hitters whose home run probabilities are considerably higher than *p*. If we replace *p* to the fourth power by the product of the four individual home run rates the players had established in their careers through 2006, the estimate changes. The probabilities were .0608, .0403, .0369, and .0324 for Ramirez, Drew, Lowell, and Varitek respectively; all were veteran players so they are based on large samples. We can use the data to get a less naïve estimate of one in (1/ [(.0608)(.0403)(.0369)(.0324)]) = 340,814. This indicates that four homers in a row by these players is about four times as likely as our first estimate. That's a big difference, but we're still off by a factor of ten from the historical data!

We now consider the difficulty from a probability modeling point of view. For a mathematician, asking "What are the chances of that?" requires that we define what "that" is. With our estimates we are computing the answer as if, when Manny Ramirez strolled to the plate, one fan turned to another and said, "What's the probability that we will see four home runs in a row right now?" Suppose instead that the same fans were having a conversation on the way into the ballpark and one asked the other, "What are the chances that we see four home runs in a row at any point during tonight's game?"

A typical major league baseball game has about 80 hitters come to the plate, so a streak of four homers has about 80 opportunities to get under way. For a rare event like this, the probability of its occurrence in 80 trials is very close to 80 times the probability that it occurs on any trial. (This logic fails for more common events because it doesn't take into account that the event occurs more than once.) Suppose we return to our first estimate for four typical major leaguers: One in 1.4 million becomes one in 1,400,000/80 or 1 in 17,500. This is now reasonably in line with the historical data.

There are lessons here for anyone hoping to use probability models. One is that we should always carefully define the event and the time frame. Another key lesson is to check, if possible, the estimate against the historically available data. Example 1 is a case in which respected academics and professionals, quoted in a leading newspaper, suggested a probability estimate that was not really incorrect, but it did not fit for the natural question. A problem about baseball is a good teaching tool and fun to consider, but we should think about how important it is to do a good job considering the probabilities of rare outside of the sports pages. Estimates of the chances of financial market collapses, nuclear power plant failures, and exceptionally strong storms, just to name a few, are key drivers of public policy.

Example 2: A streak of winless opponents

At the start of the 2009 National Football League season, the Washington Redskins had a remarkable run. Their first six games were against winless opponents! What are the chances of that?

Here is a summary of a discussion of this problem that circulated among participants in a football pool (run for entertainment purposes only, of course!) made up of academics with PhDs in a variety of subjects. One contributor suggested that we might expect the probability of six consecutive winless opponents to start the season would be about 1 in 32,768. This was determined by first computing the probability of playing six straight winless opponents, which was estimated to be 1 * (.50) * (.25) * (.125) * (.0625) * (.03125). The leading 1 appears because every team's opponent is winless in their first game. The reciprocal of the probability gives the 32,768.

This problem has many of the same features as in Example 1. A rare event is observed, and a probability is computed with an assumption of independence. As in the first example, where we replaced a home run ratio with player specific values, we can argue that the probabilities are not quite right. In fact, the Redskins week five opponent, the Carolina Panthers, had a bye week with no game in week four, which would change their probability of being winless in week five from .0625 to .125. The problem has another consideration that we should keep in mind: the event would be just as newsworthy if a team opened a season with six straight games against undefeated teams. Again, we find it hard to define the chances of an event until we have clearly defined the event.

Make the question more specific: What is the chance in any particular season that at least one National Football League team will begin playing six straight games against teams that are either all winless or all undefeated? Adding the undefeated teams and taking into account that about one quarter of the teams will play a team that has had a bye in the first six weeks makes some difference, but the most important change to the estimate of one in 32,768 comes from the fact that there are 32 teams in the league and if any of them had such a streak it would be in the news! (The 32 teams that might realize such a streak here correspond to the 80 at-bats where a home run streak might have started in Example 1.) My improved estimate is that the chance of the event is approximately once in every 400 seasons. Try your own calculation to see if you agree! The 2009 Redskins were in fact the first team to start the season with such a streak.

Problem 2: Playoff Series Length

Since 1995, major league baseball playoffs have been comprised of eight teams. In each of the American and National leagues there are three divisions: East, Central, and West. The regular season champions in the six divisions advance to the playoffs along with one wild card team (the second place team with the best record) from each league. The first round of the playoffs, called the division series, consists of four best three of five games series between the playoff teams. The remaining two rounds, the two League Championship Series and the World Series, are best four of seven games series.

When an upset occurs in the division round, it is not hard to find fans of the losing team complaining that the shorter first round series was inherently and almost criminally unfair! Sports journalists agree. This is what Darren Everson and Hannah Karp wrote in the *Wall Street Journal* on October 6, 2009:

> Nothing else in sports is as thrillingly zany—or as patently ridiculous—as the first round of Major League Baseball's playoffs.
>
> After playing nearly 14 dozen games over six months (not including spring training), the best eight teams are paired up in a fiendish creation called the Division Series, where the first team to win three games moves on. There's really no logical way to defend this practice. It's as if the Boston Marathon chose the winner by putting its top finishers through a limbo contest.

> The Division Series, which begins Wednesday, has been so volatile in recent years that the team with the better record has won just 48% of the time. When it comes to teams that win 100 games or more—the mark of an elite season—only 10 of 19 have survived. In one year, 2002, no fewer than three 100-win teams went down in the first round.
>
> "It's unfair," says Los Angeles Dodgers manager Joe Torre. "It's really Russian roulette."

There are few complaints about the fairness of seven game series. Are best of seven game series really much more fair? Is it more likely that the superior team will advance in a best of seven game series?

Suppose that teams A and B are about to meet in a playoff series and A is the superior team. Let p represent the probability that A would defeat B in a single game, so that 0.5 • p • 1. Assume the outcomes of the games are independent. Let w_5 and w_7 be the probabilities that A wins a best of five and a best of seven series. Then

$$w_5 = p^3 + 3p^3(1 - p) + 6\,p^3(1 - p)^2$$

The terms represent the probability of a three game series sweep by A, a win by A in four games and a win by A in five games respectively. The coefficient of 3 in the second term appears because there are three ways in which B might have won one game (either by winning the first, second or third game), while the 6 in the third term appears because there are six ways to arrange two wins for A and two wins for B during the first four games. For the longer series:

$$w_7 = p^4 + 4p^4(1 - p) + 10\,p^4(1 - p)^2 + 20\,p^4(1 - p)^3$$

If a seven game series is fairer than a five game series, we would expect that w_5 and w_7 would differ widely for some values of p. But they do not, as Figure 1 shows:

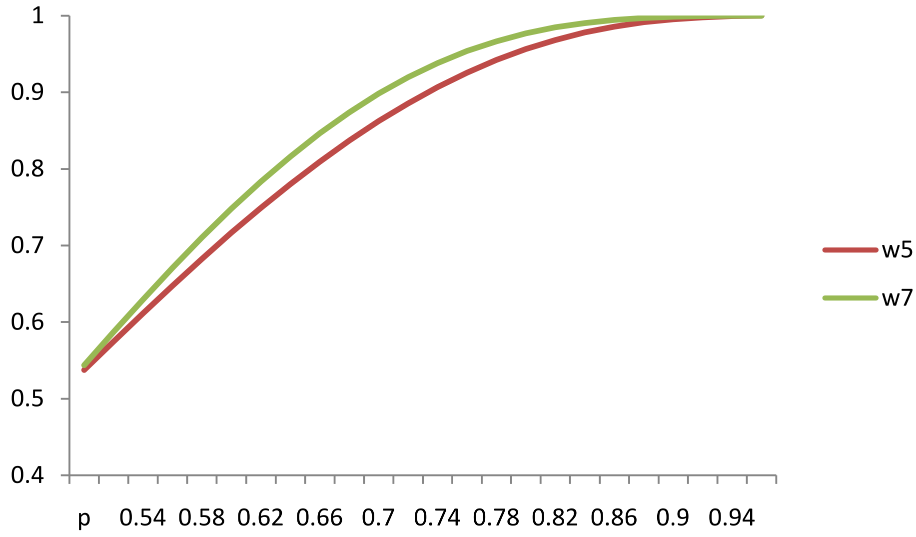

Figure 1: Probability that a team with probability *p* of winning an individual game wins a five game series (lower curve) or a seven game series (upper curve.)

The maximum difference in these two curves occurs when p is about 0.689. At that point we find that w_7 = 0.874 and w_5=0.837, a difference of only 0.037. A value of p = 0.689 is larger than we would expect in playoff games practice, and in fact it is extremely rare for a team to win 69% of its games even during the regular season.

A criticism of this model is that p is not constant because, for one thing, one of the teams will have home field advantage in each game. Also the assumption that outcomes are independent is debatable. For example, a team one loss away from elimination might use every possible strategic weapon available to win a particular game such as using a starting pitcher on short rest, or having their best relief pitcher work longer than usual. Though we can add these to our model, the curves in Figure 1 move very little.

In the two problems we have studied, people who think carefully about sports and probability suggested estimates that were wrong. The rare events were unlikely, but not fantastically unusual. Longer playoff series favor the better team, but the differences are probably less than most fans believe. Sports are a wonderful source for probability problems, and probability can shed light on sports and challenge the conventional wisdom in interesting and entertaining ways. Students learning about probability would be wise to adopt the lessons in these examples.

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