Mathematical Ranking of the Division III Track and Field Conferences

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Abstract.

In this article we provide a mathematical model for ranking of the National Collegiate Athletic Association (NCAA) Division III track and field conferences. It uses four ranking systems involving vector lengths and z-scores to measure and rank the strength of each conference using the top eight marks of each conference's championship meet. We take an average of each conference's rank to produce the final ranking. We believe that this model is superior to the current ranking system at the national meet. We conclude by using the 2008 men's outdoor season data to rank the Division III conferences for the 2009 season.

In the National Collegiate Athletic Association (NCAA), there are no ranking systems for the track and field conferences in any division. The only rankings in track and field are the school rankings at the national championship meet. Because of the way they are made, the national champion school may not even be the strongest team in its conference.

At the national meet, it is possible to win the national title with only a few top athletes. In 2008, the outdoor national title was won with a total of only 35 points. These could have been earned with three first place finishes and one fourth place finish, so the national title can be won by a school that has a single exceptional athlete. In 2008 Fisk University was tied for 29th in Division III with 10 points. Fisk University's team consisted of a single athlete who won his event at the national meet. With the ranking method used at the national meet there are conferences that do not score any points, so Fisk University's team of one athlete appears to be better than entire conferences. This is absurd. The school rankings at the national championship do not reflect the strength of a school or a conference. Rather, they show only how the top one or two athletes in each conference compare to each other. It is like ranking entire baseball teams on the basis of their best hitter and best pitcher.

People familiar with track and field form opinions about which conferences are the best based on rankings at the national meet. They assume that the conferences with the top ranking teams are the best conferences in the nation. However, if Conference A has a single All-American in every event while all the others in Conference A are mediocre while Conference B has many excellent athletes who are slightly below the level required to compete at the national championship meet, the ranking system at the national meet would make it appear that Conference A is stronger than Conference B even though were the two conferences to compete against each other in a meet, Conference A would win every event, but Conference B would easily win the meet.

This shows that the ranking method used at the national meet does not reflect the strength of the conferences. To rank conferences more accurately there needs to be a ranking system that takes into account the performances of the athletes at normal meets and not just at the national meet.

In what follows, I use vector lengths and *z*-scores to create a ranking system of the Division III track and field conferences that I believe is superior to the current system used at the national championship meet. Because athletes are usually at their best around the time of the conference meet I use their performances at the conference meet as data points.

For each event to have equal weight we need to normalize them by using ratios of a standard mark for an event and the average value for that event. In Division III there are two standard marks for each event, the national provisional mark and the national automatic mark. To qualify for the Division III National Championship Meet, athletes need to better the national provisional mark. The national championship takes a set number of athletes for the entire meet. They include all who have achieved the automatic mark and fill the remainder of the spots with athletes with the highest provisional marks. For our model we create a value without units that gives each event equal weight. Because the goal in track events is to achieve a time lower than the automatic mark, we compute the value by dividing the automatic mark by the average value for that event. For field events, the value is obtained by dividing the average value of each event by automatic mark because the goal is to exceed the automatic mark. Since these values are within a few hundredths of the ratio of automatic mark and the provisional mark they give each event equal weight.

In our model, two vectors, Vr and Vz each consisting of 20 values are employed. The vector Vr uses the ratios of the NCAA Division III automatic mark and the event averages. The vector Vz uses the event z-scores. The vector components in order are the ratios for the events: 100, 200, 400, 800, 1500, 5000, 10000, 110 hurdles, 400 hurdles, steeplechase, 4*100, 4*400, long jump, triple jump, shot put, discus, hammer, javelin, high jump, pole vault. In this model there are two ranking methods that use vector lengths. The length of the vector $(a_1, a_2, \ldots, a_{20})$ is defined as the square root of $a_1^2 + a_2^2 + \ldots + a_{20}^2$. A *z*-score is a measure of the distance in standard deviations of a sample mean to the population mean. That is, $z = \left(\overline{x} - \mu\right) / \sigma$, where $\overline{x}$ is the sample mean, μ is the population mean, and σ is the standard deviation. *z*-scores can be converted to probabilities using the standard normal curve. We are able to use *z*-scores since the results at championship meets are normally distributed.

For the second method of ranking conferences, we adapt the method that is used to score cross country meets. In cross country meets, the first place runner gets one point, the second place runner gets two points, and so on. To calculate the team score, points of the top five individuals of each team are added. The team with the lowest score wins the meet, the team with the second lowest score takes second, and so on. In cross country meets there are no ties. To break ties, the score of the teams' sixth runners are included. Whichever team's sixth runner finished higher takes the higher position. If a team has only four runners, it gets a point value for their fifth runner by using the score of the last place finisher in the meet plus one.

Our model assigns each event the same number of points and the points are totaled for each conference. As in cross country scoring, the conference with the lowest point value will be ranked number one, the conference with the second lowest point value will be ranked number two, and so on. If a conference has no entries in an event, then it gets the highest point value in the event. Since there are thirty-two conferences, every event will have the point values from one to thirty-two, so there is a total of 528 points for each event. If two conferences tie for *n*th place, we assign the average of n and $n + 1$ points to each. This keeps the number of performances throughout the conferences constant and the number of points in each event the same. If there is exactly one conference that does not have a certain event at their championship meet, that conference will be ranked last and assigned a point value of thirty two in that event. If there is more than one conference that does not have an event, we use the cross country method for breaking ties to assign point values. We call this method of counting the *modified cross country count*. We do not use the decathlon in our model because most conferences do not include it as a conference championship event.

We begin by averaging the top eight performances of all twenty events at each of the conferences' championship meet. Since twelve events are measured in units of time and eight are measured in units of distance, the averages need to be converted into a value that will give them a common unit and equal weight. It is important that the events have an equal weight to keep certain events from skewing the results. To do this, we use ratios involving the averages and the national automatic marks. Since lower times are better than the automatic mark in track events, we will divide the automatic marks by the averages for them. For field events, marks greater than the automatic mark are better so we will divide the averages by their respective automatic marks. This gives ratios with no units and equal weight that can be compared.

With these ratios, we make the vector Vr for each conference and calculate its length. We rank the conferences' vectors from longest to shortest with the conference having the longest vector length first. This gives us our first preliminary conference ranking.

We next use the z-scores and the modified cross country scoring method to assign points in each event and total them for each conference. Because faster times for running events will have a negative z-value and the slower times will have a positive z-value, we use the absolute value of the z-score in all our calculations. We use the modified z-scores to obtain the three further preliminary conference rankings. To get the overall ranking for each conference we total the points for each conference. The best conference is the one with the fewest points and the worst conference is the one with the most points. This gives us our second preliminary conference ranking.

The third ranking method ranks each conference by averaging the z-scores of each event within a conference. Ordered from largest to smallest, they give us a third preliminary conference ranking from best to worst.

For the fourth ranking method, we convert the *z*-scores into their corresponding probabilities from the normal curve. When a conference does not have an event, we assign that conference a probability of zero for that event. We then use these probabilities to create a twenty-component vector, Vz, for each conference using the same event order as used for Vr. Our fourth preliminary ranking of the conferences is by the lengths of the Vz vectors, from longest to shortest.

To produce our final conference ranking we average the four preliminary rankings. We conclude by applying our model to produce a ranking system of the NCAA Division III track and field conferences for the 2008 outdoor season. By employing four preliminary rankings, we were able to use a variety of methods to rank the conferences. Even though some of the conferences differed in their position from method to method, they were not far apart. In fact, there were two conferences whose position was exactly the same through every ranking. In the method that used the modified cross country count, nineteen of the thirty-two conferences had the same ranking as the final ranking, including the bottom eight.

Although some of the conferences near the top of our final ranking are also near the top of the national championship ranking there were some significant differences. This demonstrates that the scores at the national championship meet reflect only the strength of the top few athletes in a conference rather than the overall strength of the conference.

Our second preliminary ranking gives an added bonus. It ranks every conference by event which allows us to group certain events together to see where conferences rank based only on those events. This enables us to see which conference is the best sprinting conference, distance conference, field event conference, and so on. Also, it allows athletes who are curious as to where their conference ranks nationally based only on their events a way to see where their conference ranks by adding the points for each conference in just those events.

Our assumption in our model is that athletes have near peak performances in their conference championship. Weather conditions and other factors may affect an athlete's performance, as can things such as injuries, false starts, and disqualifications, especially for top athletes.

In the following tables conferences are identified by their usual abbreviations.

2008 Ranking of Division III Track and Field Conferences

Z-Score Count		
1st	**MIAC**	1.00
2nd	**WIAC**	2.00
3rd	**IIAC**	3.00
4th	**CCIW**	4.00
5th	**NESCAC**	5.00
6th	**OAC**	6.00
7th	**UAA**	7.00
8th	**SUNYAC**	8.00
9th	**NWC**	9.00
10th	**MWC**	10.00
11th	**MIAA**	11.00
12th	**SCIAC**	12.00
13th	**CC**	13.00
14th	**NEWMAC**	14.00
15th	**NJAC**	15.00
16th	**MAC**	16.00
17th	**ODAC**	17.00
18th	**PAC**	18.00
19th	**HCAC**	19.00
20th	**LC**	20.00
21st	**USAS**	21.00
22nd	**NCAC**	22.00
23rd	**ASC**	23.00
24th	**SCAC**	24.00
25th	**CAC**	25.00
26th	**Empire 8**	26.00
27th	**LEC**	27.00
28th	**LLC**	28.00
29th	**NAC**	29.00
30th	**MSCAC**	30.00
31st	**CUNYC**	31.00
32nd	**UMAC**	32.00

Ratio Vector Lengths		
1st	**WIAC**	1.00
2nd	**MIAC**	2.00
3rd	**IIAC**	3.00
4th	**NESCAC**	4.00
5th	**NWC**	5.00
6th	**OAC**	6.00
7th	**MWC**	7.00
8th	**SUNYAC**	8.00
9th	**UAA**	9.00
10th	**NEWMAC**	10.00
11th	**CCIW**	11.00
12th	**CC**	12.00
13th	**MIAA**	13.00
14th	**NJAC**	14.00
15th	**MAC**	15.00
16th	**SCIAC**	16.00
17th	**HCAC**	17.00
18th	**SCAC**	18.00
19th	**LC**	19.00
20th	**PAC**	20.00
21st	**USAS**	21.00
22nd	**ODAC**	22.00
23rd	**CAC**	23.00
24th	**NAC**	24.00
25th	**LEC**	25.00
26th	**MSCAC**	26.00
27th	**Empire 8**	27.00
28th	**LLC**	28.00
29th	**ASC**	29.00
30th	**UMAC**	30.00
31st	**NCAC**	31.00
32nd	**CUNYC**	32.00

Z-Score Averages		
1st	**WIAC**	1.00
2nd	**MIAC**	2.00
3rd	**IIAC**	3.00
4th	**CCIW**	4.00
5th	**NESCAC**	5.00
6th	**OAC**	6.00
7th	**SUNYAC**	7.00
8th	**NWC**	8.00
9th	**UAA**	9.00
10th	**MWC**	10.00
11th	**SCIAC**	11.00
12th	**MIAA**	12.00
13th	**CC**	13.00
14th	**NEWMAC**	14.00
15th	**NJAC**	15.00
16th	**MAC**	16.00
17th	**ODAC**	17.00
18th	**PAC**	18.00
19th	**HCAC**	19.00
20th	**LC**	20.00
21st	**SCAC**	21.00
22nd	**USAS**	22.00
23rd	**Empire 8**	23.00
24th	**ASC**	24.00
25th	**NCAC**	25.00
26th	**CAC**	26.00
27th	**LLC**	27.00
28th	**LEC**	28.00
29th	**NAC**	29.00
30th	**MSCAC**	30.00
31st	**UMAC**	31.00
32nd	**CUNYC**	32.00

Z-Score Vector Lengths		
1st	**WIAC**	1.00
2nd	**MIAC**	2.00
3rd	**IIAC**	3.00
4th	**CCIW**	4.00
5th	**NESCAC**	5.00
6th	**OAC**	6.00
7th	**SUNYAC**	7.00
8th	**NWC**	8.00
9th	**UAA**	9.00
10th	**MWC**	10.00
11th	**SCIAC**	11.00
12th	**MIAA**	12.00
13th	**NEWMAC**	13.00
14th	**NJAC**	14.00
15th	**CC**	15.00
16th	**MAC**	16.00
17th	**HCAC**	17.00
18th	**PAC**	18.00
19th	**ODAC**	19.00
20th	**NCAC**	20.00
21st	**LC**	21.00
22nd	**USAS**	22.00
23rd	**SCAC**	23.00
24th	**ASC**	24.00
25th	**Empire 8**	25.00
26th	**CAC**	26.00
27th	**LLC**	27.00
28th	**LEC**	28.00
29th	**NAC**	29.00
30th	**MSCAC**	30.00
31st	**CUNYC**	31.00
32nd	**UMAC**	32.00

Final Ranking

Rank	Conference	Average
1st	**WIAC**	1.25
2nd	**MIAC**	1.75
3rd	**IIAC**	3.00
4th	**NESCAC**	4.75
5th	**CCIW**	5.75
6th	**OAC**	6.00
7th	**NWC**	7.33
8th	**SUNYAC**	7.50
9th	**UAA**	8.50
10th	**MWC**	9.25
11th	**MIAA**	12.00
12th	**SCIAC**	12.50
13th	**NEWMAC**	12.75
14th	**CC**	13.25
15th	**NJAC**	14.50
16th	**MAC**	15.67
17th	**HCAC**	18.00
18th	**PAC**	18.50
19th	**ODAC**	18.75
20th	**LC**	20.00
T-21st	**USAS**	21.50
T-21st	**SCAC**	21.50
23rd	**NCAC**	24.50
24th	**ASC**	25
25th	**CAC**	25.00
26th	**Empire 8**	25.25
27th	**LEC**	27
28th	**LLC**	27.5
29th	**NAC**	27.75
30th	**MSCAC**	29.00
31st	**CUNYC**	31.25
32nd	**UMAC**	31.5

NCAA Championship Rank

Rank	Conference	Points
1st	**WIAC**	97
2nd	**IIAC**	70
3rd	**CCIW**	54.5
4th	**SUNYAC**	54
5th	**MIAC**	52
6th	**NESCAC**	49
7th	**NWC**	42
8th	**OAC**	37
9th	**ASC**	35
10th	**UAA**	34
11th	**USAS**	27
12th	**MAC**	26
13th	**MWC**	25
14th	**LC**	24
T-15th	**MIAA**	23
T-15th	**NJAC**	23
17th	**Empire 8**	20
18th	**PAC**	19
19th	**ODAC**	18
20th	**SCIAC**	12
21st	**SCIAC**	11
22nd	**HCAC**	9.5
23rd	**NCAC**	9
24th	**SCAC**	8.5
25th	**NAC**	8
26th	**NEWMAC**	5.5
27th	**CUNYC**	4
28th	**CC**	3
29th	**LEC**	1
T-30th	**CAC**	0
T-30th	**LLC**	0
T-30th	**UMAC**	0

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