Biodiversity, Mathematics and Sustainability: Maintaining the Planet's Biotic Resources

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Our planet is replete with living systems that provide the essential infrastructure for human civilization. All life processes on the planet rely upon the photosynthetic capacity of aquatic and terrestrial plants and microbes to produce the high-energy carbon compounds upon which all non-photosynthetic organisms depend for their energetic needs. The complex, interwoven linkages between diverse organisms from numerous taxa have arisen via biological evolution over a long time span relative to the recent advent of human civilization, yet the direct actions and indirect influences of humanity have created great strains on the planetary biota. Mathematical methods are one of the key tools used to assess the current impacts of human actions, project how modifications of anthropogenic forcing might affect living systems over the future, and suggest hypotheses to assist in teasing apart the interactions and feedbacks between environment and biotic systems to better inform science and public policy. These are all key questions addressed by sustainability science in determining how we might appropriately modify human actions and impacts in order to sustain and support human societies and the living systems upon which they depend.

In common parlance, ***biodiversity*** refers to the collection of species present in a region, but in practice it includes the variety of life present at a location including the connections and systems of which the species are a part. Mathematical methods have been used to characterize components of biodiversity, as a means of determining whether these are changing at a location, comparing these across locations, and evaluating hypotheses about the factors that affect biodiversity. The simplest metric used is ***species richness*** which is simply a count of the number of species, typically from some restricted set of taxa such as plants or vertebrates, present at a location. This provides some useful information to compare locations, but does not account for the differences in abundance of the species present. A location with a very large number of individuals or biomass of a single species and only a few individuals of other species is quite different from a location with about equivalent numbers of all the species present. If many species at a location are rarely occurring, then a major disturbance such as a fire could potentially greatly reduce the biodiversity at the location quickly. In this sense, a location with higher ***evenness*** of distribution of species abundances is more resilient.

Diversity indices are metrics that can account for the distribution of abundances across taxa, incorporating aspects of both richness and evenness. Two common indices are the Shannon index $H = -\sum_{i=1}^{S} p_i \log p_i$ where S is the number of species present and p_i is the fraction of all individuals present which are of species I, and the Simpson index $I = 1 - \sum_{i=1}^{S} p_i^2$. These are not perfect metrics however, since they do not incorporate differences between species other than abundances, and they don't readily handle the different spatial scales at which diversity is considered (e.g. as you move from a single acre plot, to an entire management region of many thousands of acres). Conservation biologists use a mixture of indices such as these however to evaluate alternative management, such as harvesting, stocking and translocation of species in a region.

While there is an inferred value in preserving biodiversity since human societies depend upon, and benefit directly from, living systems, there is also an argument for maintaining biodiversity due to it's intrinsic value independent of its benefit to humanity. The term ***ecosystem services*** refers in general to the benefits provided to human society from biodiversity including resources such as food, regulating services such as pollination and flood control, and support processes such as nutrient cycling. Current questions in conservation biology often concern how to adequately quantify these ecosystem services, in order to consider the costs and potential benefits of alternative management or public policies. The mathematical methods used to quantify ecosystem services take into account the feedbacks between human actions and the living system response, the changes through time and space (called the ***system dynamics***) that arise when different actions are taken, and the uncertainty of future conditions such as climate and in our ability to account for the complexity of the systems affected.

One of the key questions addressed by mathematical methods in conservation biology concerns ***extinction risks***. The objective is to determine the chance that, over some time period, a population (or possibly an entire species) goes extinct, and how this chance would be affected by some action. At the level of local populations, this is typically addressed using ***population viability analysis (PVA)*** which accounts for the demographics (e.g. size and age structure) of a population, and determines from estimates of birth and death rates what the probability is that the population will remain above some threshold level for some time period in the future. The mathematical methods involved include matrix algebra, to account for the structure of the population, statistics to estimate birth and death rates from data, and probability theory to project how uncertainties in demography or environmental factors affect the chance the population persists over the time period of concern. PVA is also helpful in identifying threatened and endangered species and determining what kinds of monitoring schemes are needed to improve the viability of such populations.

Analyzing the spatial distribution of biota across the planet is crucial to identifying important habitat for species of particular concern and to project the impact of climate change and human actions on where species can persist. As invasive species can be a major factor affecting biodiversity, maps of species distributions also provide a means to evaluate the extent of invasion, locate potentially damaging pathogen and pest outbreaks and investigate the benefits of actions to control disturbances such as fire. Geographic information systems (GIS) are critical tools in the spatial analysis of biodiversity, and these tools rely heavily upon mathematical manipulations of data in a spatial context. Species distribution models provide a mathematical means to predict the presence and absence of species in regions depending upon environmental conditions, and have been a critical tool to analyze potential shifts in species ranges under diverse climate change scenarios.

I have only touched here on the many ways that mathematics has been applied to investigate biodiversity and its relationship to ensuring sustainability of human civilization. The references provide some additional examples, but I wish to encourage appreciation of the fact that there are numerous open challenges, many ways that quantitatively adept individuals can contribute to sustainability science, and a need to connect the mathematics to data that often can only be obtained from a mixture of remote observation (e.g. from satellites) and on-the-ground field efforts.

For further reading:

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