Math and Global Temperature

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&
Math and Climate
Research Network

Thanks to:
many friends
& colleagues
Evidence of a Changing Climate

Intergovernmental Panel on Climate Change (IPCC):
Warming is "unequivocal"

Dots: yearly average
Curve: decadal average
Blue: uncertainty interval
Why do we try to predict?

Warming is NOT uniform
Why use Mathematical Models?

1

many

C. Jones
Simplest Energy Balance Model

Energy gains equal Energy losses

CIMSS/ U Wisc
Simplest Energy Balance Model

PLAN: Use this model to understand the role of greenhouse gases in regulating our temperature
Understanding Radiated Energy

Black Body Radiation - Planck

Spectral Intensity vs Wavelength

For black bodies at different temperatures (0°C=273K)

Sun: ~6000K, emits mainly in ultraviolet (UV) and visible spectrum (shortwave)
Earth: ~300K, emits mainly in infrared (IR) spectrum (longwave)

Total energy flux over all wavelengths for temp T is \( \sigma T^4 = (5.67 \times 10^{-8})T^4 \) -- Stefan-Boltzmann
Earth’s Average Temperature

Question: What is Earth’s average global annual temperature $T$? What do you think?
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Answer: $15^\circ$C ($59^\circ$F)
Earth’s Average Temperature

**Question:** What is Earth’s average global annual temperature $T_e$? What do you think?

**Answer:** $15^\circ$C ($59^\circ$F)

**Question:** Why? What controls it?
Earth’s Average Temperature

Question: What is Earth’s average global annual temperature $T_e$? What do you think?
Answer: 15°C (59°F)

Question: Why? What controls it?
Answer: Lots of things!
Earth’s Average Temperature

**Question:** What is Earth’s average global annual temperature $T_e$? What do you think?

**Answer:** 15°C (59°F)

**Question:** Why? What controls it?

**Answer:** Lots of things!

**Let’s use math to test this hypothesis:** Earth’s temperature is a consequence of

1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

So we pretend the atmosphere is transparent to radiation: “no atmosphere”
Hypothesis: Earth’s temperature is a consequence of

1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)
Hypothesis: Earth’s temperature is a consequence of
1. Blackbody radiation
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4. Albedo (reflectivity)

Question: Why are the rays from the sun approx. parallel? Which hypothesis?
Energy Balance, No Atmosphere

**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Question:** Why are the rays from the sun approx. parallel? Which hypothesis?

**Answer:** Sun is so far away (hypothesis 2)
**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Question:** What’s the cross sectional area of the Earth (radius r)? Which hypothesis?
Hypothesis: Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Question: What’s the cross sectional area of the Earth (radius r)? Which hypothesis?
Answer: Area = \( \pi r^2 \) m\(^2\) (hypothesis 3)
**Energy Balance, No Atmosphere**

**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Question:** What’s the surface area of the Earth (radius r)? Which hypothesis?
**Energy Balance, No Atmosphere**

**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Question:** What’s the surface area of the Earth (radius r)? Which hypothesis?

**Answer:** Surface Area = $4\pi r^2$ (hypothesis 3)
**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Question:** What’s the terrestrial flux? Which hypothesis?

- **Terrestrial flux**
  - Surface Area = \(4\pi r^2\) m\(^2\)
Figuring Out Terrestrial Flux

Black Body Radiation - Planck

Spectral Intensity vs Wavelength

For black bodies at different temperatures (0°C=273K)

Sun: ~6000K, emits mainly in ultraviolet (UV) and visible spectrum (shortwave)
Earth: ~300K, emits mainly in infrared (IR) spectrum (longwave)

Total energy flux over all wavelengths for temp T is \( \sigma T^4 = (5.67 \times 10^{-8})T^4 \) -- Stefan-Boltzmann
Hypothesis: Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Question: What’s the terrestrial flux? Which hypothesis?

Answer: Terrestrial flux = (5.67x10^{-8})T_e^4  Wm^{-2} (hyp. 1)
Energy Balance, No Atmosphere

**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Question:** What’s the Solar flux? Which hypothesis?
**Answer:** Solar flux $S = 1367 \text{ Wm}^{-2}$ (hypothesis 1,2)

Solar flux $1367 \text{ Wm}^{-2}$
Rays approx. parallel

Terrestrial flux $= (5.67 \times 10^{-8})T_e^4$
Surface Area $= 4\pi r^2 \text{ m}^2$
Energy Balance, No Atmosphere

**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Question:** What fraction of solar radiation does the earth reflect? What hypothesis?

Solar flux $1367 \text{ Wm}^{-2}$
Rays approx. parallel

Terrestrial flux $= (5.67 \times 10^{-8})T_e^4$
Surface Area $= 4\pi r^2 \text{ m}^2$

Area $= \pi r^2 \text{ m}^2$
**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Question:** What fraction of solar radiation does the earth reflect? What hypothesis?

**Examples:**
- Fresh snow reflects 85%
- Sandy desert reflects 40%
- Grasslands reflect 18%
- Cities reflect 16%
- Ocean reflects 6%
**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Question:** What fraction of solar radiation does the earth reflect? What hypothesis?

**Answer:** Planetary average albedo 0.3 (hyp. 4)

**Examples:**
- Fresh snow reflects 85%
- Sandy desert reflects 40%
- Grasslands reflect 18%
- Cities reflect 16%
- Ocean reflects 6%

\[
\text{Solar flux } 1367 \text{ Wm}^{-2}
\]

\[
\text{Terrestrial flux } = (5.67 \times 10^{-8})T_e^4
\]

\[
\text{Surface Area} = 4\pi r^2 \text{ m}^2
\]
Energy Balance, No Atmosphere

Hypothesis: Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

OK! Now we understand the picture,
Let’s do energy balance...

\[ \text{Energy gains} = \text{Energy losses} \]

**Energy Gains**

**Energy Losses**

\[
\begin{align*}
\text{Solar flux} & = 1367 \text{ Wm}^{-2} \\
\text{Albedo} & = 0.3
\end{align*}
\]

Terrestrial flux = \((5.67 \times 10^{-8})T_e^4\)
Surface Area = \(4\pi r^2 \text{ m}^2\)
Energy Balance, No Atmosphere

Hypothesis: Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Energy Gain
Solar flux * cross-sect area = ???

Energy gains
Energy losses

Energy gains
Energy losses

Solar flux 1367 Wm$^{-2}$
Rays approx. parallel
Albedo 0.3

Terrestrial flux = (5.67x10$^{-8}$)T$^4$
Surface Area = 4πr$^2$ m$^2$
Energy Balance, No Atmosphere

Hypothesis: Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Energy Gain
Solar flux * cross-sect area = 1367 Wm\(^{-2}\) * \(\pi r^2\) m\(^2\) = 1367\(\pi r^2\) W

Terrestrial flux = \((5.67 \times 10^{-8})T_e^4\)
Surface Area = \(4\pi r^2\) m\(^2\)
**Energy Balance, No Atmosphere**

**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Energy Gain**
Solar flux * cross-sect area = 1367πr^2 W

**Energy loss by Terrestrial Radiation**
Terrestrial flux * surface area = ???

**Energy gains equal Energy losses**

**Energy flux**
Solar flux 1367 Wm^{-2}
Rays approx. parallel

Terrestrial flux = (5.67x10^{-8})T_e^4
Surface Area = 4πr^2 m^2

Area = πr^2 m^2
Hypothesis: Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Energy Gain
Solar flux * cross-sect area = 1367πr² W

Energy loss by Terrestrial Radiation
Terrestrial flux * surface area = (5.67x10⁻⁸)T_e⁴ 4πr² W
**Energy Balance, No Atmosphere**

**Hypothesis**: Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Energy Gain**
Solar flux * cross-sect area = 1367πr² W

**Energy loss by Terrestrial Radiation**
Terrestrial flux * surface area = (5.67x10⁻⁸)T⁴ 4πr² W

**Energy loss by Albedo**
Reflected solar radiation = ???

Solar flux 1367 Wm⁻²
Rays approx. parallel

Terrestrial flux = (5.67x10⁻⁸)Tₑ⁴
Surface Area = 4πr² m²

Area = πr² m²
**Hypothesis:** Earth’s temperature is a consequence of:

1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Energy Gain**
Solar flux * cross-sect area = $1367\pi r^2$ W

**Energy loss by Terrestrial Radiation**
Terrestrial flux * surface area = $(5.67 \times 10^{-8})T^4 \times 4\pi r^2$ W

**Energy loss by Albedo**
Reflected solar radiation = $0.3 \times 1367 \pi r^2$ W
Energy Balance, No Atmosphere

**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Energy Gain**
Solar flux * cross-sect area = 1367πr² W

**Energy loss by Terrestrial Radiation**
Terrestrial flux * surface area = (5.67x10⁻⁸)T⁴ 4πr² W

**Energy loss by Albedo**
Reflected solar radiation = 410πr² W
Hypothesis: Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Energy Gain
Solar flux * cross-sect area = $1367\pi r^2$ W

Energy loss by Terrestrial Radiation
Terrestrial flux * surface area = $(5.67 \times 10^{-8})T^4 4\pi r^2$ W

Energy loss by Albedo
Reflected solar radiation = $410\pi r^2$ W

Balance
$1367\pi r^2 = 410\pi r^2 + (5.67 \times 10^{-8})T^4 4\pi r^2$
Energy Balance, No Atmosphere

**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Energy Gain**
Solar flux * cross-sect area = $1367\pi r^2$ W

**Energy loss by Terrestrial Radiation**
Terrestrial flux * surface area = $(5.67 \times 10^{-8})T^4 4\pi r^2$ W

**Energy loss by Albedo**
Reflected solar radiation = $410\pi r^2$ W

**Balance**
$1367\pi r^2 = 410\pi r^2 + (5.67 \times 10^{-8})T^4 4\pi r^2$
$1367 = 410 + 4(5.67 \times 10^{-8})T^4$
**Energy Balance, No Atmosphere**

**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Energy Gain**
Solar flux * cross-sect area = $1367\pi r^2$ W

**Energy loss by Terrestrial Radiation**
Terrestrial flux * surface area = $(5.67\times10^{-8})T^4 4\pi r^2$ W

**Energy loss by Albedo**
Reflected solar radiation = $410\pi r^2$ W

**Balance**

$1367\pi r^2 = 410\pi r^2 + (5.67\times10^{-8})T^4 4\pi r^2$

$1367 = 410 + 4(5.67\times10^{-8})T^4$

$(5.67\times10^{-8})T^4 = 239$
Hypothesis: Earth’s temperature is a consequence of
   1. Blackbody radiation
   2. Distance from Sun
   3. Size
   4. Albedo (reflectivity)

Energy Gain
Solar flux * cross-sect area = 1367πr² W

Energy loss by Terrestrial Radiation
Terrestrial flux * surface area = (5.67x10⁻⁸)T⁴ 4πr² W

Energy loss by Albedo
Reflected solar radiation = 410πr² W

Balance
1367πr² = 410πr² + (5.67x10⁻⁸)T⁴ 4πr²
1367 = 410 + 4(5.67x10⁻⁸)T⁴
(5.67x10⁻⁸)T⁴ = 239
T = 239^(1/4)(5.67x10⁻⁸)^(-1/4)
Energy Balance, No Atmosphere

**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Energy Gain**
Solar flux * cross-sect area = 1367πr²  W

**Energy loss by Terrestrial Radiation**
Terrestrial flux * surface area = (5.67x10⁻⁸)T⁴ 4πr²  W

**Energy loss by Albedo**
Reflected solar radiation = 410πr²  W

**Balance**
1367πr² = 410πr² + (5.67x10⁻⁸)T⁴ 4πr²
1367 = 410 + 4(5.67x10⁻⁸)T⁴
(5.67x10⁻⁸)T⁴ = 239
T = 239⁽¹/⁴⁾(5.67x10⁻⁸)(⁻¹/⁴)
T = 3.93x64.8
Energy Balance, No Atmosphere

Hypothesis: Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Energy Gain
Solar flux * cross-sect area = 1367πr^2  W

Energy loss by Terrestrial Radiation
Terrestrial flux * surface area = (5.67x10^-8)T^4 4πr^2  W

Energy loss by Albedo
Reflected solar radiation = 410πr^2  W

Balance
1367πr^2 = 410πr^2 + (5.67x10^-8)T^4 4πr^2
1367 = 410 + 4(5.67x10^-8)T^4
(5.67x10^-8)T^4 = 239
T = 239^(1/4)(5.67x10^-8)^(-1/4)
T = 3.93x64.8

So.......
T = 3.93x64.8 = 255K
Energy Balance, No Atmosphere

**Hypothesis:** Earth’s temperature is a consequence of

1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Energy Gain**
Solar flux * cross-sectional area = $1367\pi r^2$ W

**Energy loss by Terrestrial Radiation**
Terrestrial flux * surface area = $(5.67 \times 10^{-8}) T^4 \ 4\pi r^2$ W

**Energy loss by Albedo**
Reflected solar radiation = $410\pi r^2$ W

**Balance**

\[
1367\pi r^2 = 410\pi r^2 + (5.67 \times 10^{-8}) T^4 \ 4\pi r^2
\]

\[
1367 = 410 + 4(5.67 \times 10^{-8}) T^4
\]

\[
(5.67 \times 10^{-8}) T^4 = 239
\]

\[
T = 239^{(1/4)}(5.67 \times 10^{-8})^{-\frac{1}{4}}
\]

\[
T = 3.93 \times 64.8
\]

So....... 

\[
T = 3.93 \times 64.8 = 255 K = ???^\circ C
\]

(Remember $273 K = 0^\circ C$)
Hypothesis: Earth’s temperature is a consequence of:
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Energy Gain
Solar flux * cross-sect area = 1367πr² W

Energy loss by Terrestrial Radiation
Terrestrial flux * surface area = (5.67×10⁻⁸)T⁴ 4πr² W

Energy loss by Albedo
Reflected solar radiation = 410πr² W

Balance
1367πr² = 410πr² + (5.67×10⁻⁸)T⁴ 4πr²
1367 = 410 + 4(5.67×10⁻⁸)T⁴
(5.67×10⁻⁸)T⁴ = 239
T = 239^(1/4)(5.67×10⁻⁸)^(-1/4)
T = 3.93×64.8

So......
T = 3.93×64.8 = 255K = -18°C

Does that sound about right?
Energy Balance, No Atmosphere

**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Energy Gain**
Solar flux * cross-sect area = 1367πr² W

**Energy loss by Terrestrial Radiation**
Terrestrial flux * surface area = (5.67x10⁻⁸)T⁴ 4πr² W

**Energy loss by Albedo**
Reflected solar radiation = 410πr² W

**Balance**
1367πr² = 410πr² + (5.67x10⁻⁸)T⁴ 4πr²
1367 = 410 + 4(5.67x10⁻⁸)T⁴
(5.67x10⁻⁸)T⁴ = 239
T = 239⁽¹/⁴⁾(5.67x10⁻⁸)(⁻¹/⁴)
T = 3.93×64.8

**So.......**
T = 3.93×64.8 = 255K = -18°C

*Does that sound about right?*
*Too Cold – Snowball Earth!*
Hypothesis: Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Energy Gain
Solar flux * cross-sect area = 1367πr² W

Energy loss by Terrestrial Radiation
Terrestrial flux * surface area = (5.67x10⁻⁸)T⁴ 4πr² W

Energy loss by Albedo
Reflected solar radiation = 410πr² W

Balance
1367πr² = 410πr² + (5.67x10⁻⁸)T⁴ 4πr²
1367 = 410 + 4(5.67x10⁻⁸)T⁴
(5.67x10⁻⁸)T⁴ = 239
T = 239^(1/4) (5.67x10⁻⁸)^(-1/4)
T = 3.93 x 64.8

So......
T = 3.93 x 64.8 = 255K = -18°C

Does that sound about right?
Too Cold – Snowball Earth!
Recall: our current global average surface temp is 15°C
Energy Balance, No Atmosphere

Hypothesis: Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

Conclusions:
With these hypotheses, surface temp is 33°C too low. We need to include atmosphere to correct this.

255K is a good estimate for the top of the atmosphere, Which does satisfy the hypotheses more closely.

So......
T = 3.93×64.8 = 255K = -18°C

Does that sound about right?
Too Cold – Snowball Earth!
Recall: our current global average surface temp is 15°C
**Energy Balance, No Atmosphere**

**Hypothesis:** Earth’s temperature is a consequence of
1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)

**Conclusions:**
With these hypotheses, surface temp is 33°C too low. We need to include atmosphere to correct this.

255K is a good estimate for the top of the atmosphere, which does satisfy the hypotheses more closely.

Need new hypotheses about absorption of solar and terrestrial radiation by the atmosphere.

So.......

\[ T = 3.93 \times 64.8 = 255K = -18°C \]

Does that sound about right?
Too Cold – Snowball Earth!
Recall: our current global average surface temp is 15°C
Greenhouse Gases (GHGs)
Atmospheric Absorption Spectrum
Hypotheses: Earth’s temperature is a consequence of
1. Radiation, distance from sun, albedo
2. Atmosphere transparent to solar radiation
3. Atmosphere absorbs some terrestrial radiation

How do these assumptions affect our energy balance computation?
Energy Balance, GHGs in Atmosphere

Hypotheses: Earth’s temperature is a consequence of
1. Radiation, distance from sun, albedo
2. Atmosphere transparent to solar radiation
3. Atmosphere absorbs some terrestrial radiation

How do these assumptions affect our energy balance computation?

Energy Gain - unchanged by new assumptions
Solar flux * cross-sect area = 1367πr²
Energy Balance, GHGs in Atmosphere

**Hypotheses:** Earth’s temperature is a consequence of
1. Radiation, distance from sun, albedo
2. Atmosphere transparent to solar radiation
3. Atmosphere absorbs some terrestrial radiation

How do these assumptions affect our energy balance computation?

**Energy Gain - unchanged by new assumptions**
Solar flux * cross-sect area = 1367πr^2

**Energy loss by Albedo – unchanged**
Reflected solar radiation = 410πr^2 W
<table>
<thead>
<tr>
<th>Hypotheses: Earth’s temperature is a consequence of</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Radiation, distance from sun, albedo</td>
</tr>
<tr>
<td>2. Atmosphere transparent to solar radiation</td>
</tr>
<tr>
<td>3. Atmosphere absorbs some terrestrial radiation</td>
</tr>
</tbody>
</table>

How do these assumptions affect our energy balance computation?

**Energy Gain - unchanged by new assumptions**
Solar flux * cross-sect area = \(1367\pi r^2\)

**Energy loss by Albedo – unchanged**
Reflected solar radiation = \(410\pi r^2\) W

**Energy loss by Terrestrial Radiation - reduced! Let’s use parameter \(\alpha\) to reduce it.**
\(\alpha(\text{Terrestrial flux} \times \text{surface area}) = \alpha(5.67 \times 10^{-8})T^4 4\pi r^2\)
Hypotheses: Earth’s temperature is a consequence of
1. Radiation, distance from sun, albedo
2. Atmosphere transparent to solar radiation
3. Atmosphere absorbs some terrestrial radiation

How do these assumptions affect our energy balance computation?

Energy Gain - unchanged by new assumptions
Solar flux * cross-sect area = $1367\pi r^2$

Energy loss by Albedo – unchanged
Reflected solar radiation = $410\pi r^2$ W

Energy loss by Terrestrial Radiation - reduced! Let’s use parameter $\alpha$ to reduce it.
$\alpha$(Terrestrial flux * surface area) = $\alpha(5.67 \times 10^{-8})T^4 \times 4\pi r^2$

New Balance
$1367\pi r^2 = 410\pi r^2 + \alpha(5.67 \times 10^{-8})T^4 \times 4\pi r^2$

$T = \frac{255}{\alpha^{1/4}}$
Energy Balance, GHGs in Atmosphere

\[ T = \frac{255}{\alpha^{1/4}} \]

**Hypotheses:** Earth’s temperature is a consequence of
1. Radiation, distance from sun, albedo
2. Atmosphere transparent to solar radiation
3. Atmosphere absorbs some terrestrial radiation

**Case \( \alpha=1 \):** Corresponds to no greenhouse gases
Atmosphere is transparent to terr. radiation, &
\[ T = 255 \, \text{K} = -18^\circ \text{C}, \text{as before} \]
Snowball Earth! We need *some* GHGs to avoid this.

As greenhouse gases increase: \( \alpha \) does what??? Increase or decrease?
Energy Balance, GHGs in Atmosphere

**Hypotheses:** Earth’s temperature is a consequence of
1. Radiation, distance from sun, albedo
2. Atmosphere transparent to solar radiation
3. Atmosphere absorbs some terrestrial radiation

New Energy Balance: \[ T = \frac{255}{\alpha^{1/4}} \]

Case \( \alpha=1 \): Corresponds to no greenhouse gases
Atmosphere is transparent to terr. radiation, &
\[ T = 255 \text{ K} = -18^\circ \text{C}, \text{ as before} \]
Snowball Earth! We need *some* GHGs to avoid this.

As greenhouse gases increase: \( \alpha \) decreases
So predicted temperature \( T \) does what ??? Increase or decrease?
Energy Balance, GHGs in Atmosphere

**Hypotheses:** Earth’s temperature is a consequence of
1. Radiation, distance from sun, albedo
2. Atmosphere transparent to solar radiation
3. Atmosphere absorbs some terrestrial radiation

**New Energy Balance:** \[ T = \frac{255}{\alpha^{1/4}} \]

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Snowball Earth! We need *some* GHGs to avoid this.

**As greenhouse gases increase: \( \alpha \) decreases**
So predicted temperature \( T \) increases
Energy Balance, GHGs in Atmosphere

**Hypotheses:** Earth’s temperature is a consequence of
1. Radiation, distance from sun, albedo
2. Atmosphere transparent to solar radiation
3. Atmosphere absorbs some terrestrial radiation

**New Energy Balance:** \( T = \frac{255}{\alpha^{1/4}} \)

**Case \( \alpha=1: \) Corresponds to no greenhouse gases**
Atmosphere is transparent to terr. radiation,
\( T = 255 \text{ K} = -18^\circ \text{C}, \) as before
Snowball Earth! We need *some* GHGs to avoid this.

As greenhouse gases increase: \( \alpha \) decreases
So predicted temperature \( T \) increases

**Conclusion**
As greenhouse gases keep increasing, temperature keeps increasing...
Energy Balance, GHGs in Atmosphere

Hypotheses: Earth’s temperature is a consequence of
1. Radiation, distance from sun, albedo
2. Atmosphere transparent to solar radiation
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New Energy Balance: \[ T = \frac{255}{\alpha^{1/4}} \]

Case \( \alpha=1 \): Corresponds to no greenhouse gases
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Snowball Earth! We need \textit{some} GHGs to avoid this.

As greenhouse gases increase: \( \alpha \) decreases
So predicted temperature \( T \) increases

Conclusion
As greenhouse gases keep increasing, temperature keeps increasing...
To Learn More...

www.mathclimate.org
Mathematics and Climate Research Network – Education pages

www.mpe2013.org
Mathematics of Planet Earth 2013 – Education pages

www.mathaware.org
Math Awareness Month 2013 – Mathematics and Sustainability

www.dimacs.rutgers.edu/MPE
DIMACS Mathematics and Sustainability Curriculum Modules
And More...

John Marshall and Alan Plumb (MIT)

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THANK YOU!

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